Dark energy problem: from phantom theory to modified Gauss–Bonnet gravity

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Abstract
The solution of the dark energy problem in models without scalars is presented. It is shown that a late-time accelerating cosmology may be generated by an ideal fluid with some implicit equation of state. The evolution of the universe within modified Gauss–Bonnet gravity is considered. It is demonstrated that such a gravitational approach may predict the (quintessential, cosmological constant or transient phantom) acceleration of the late-time universe with a natural transition from deceleration to acceleration (or from non-phantom to phantom era in the last case).

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1. Introduction

According to recent astrophysical data, the current universe is expanding with acceleration. Such accelerated behaviour of the universe is supposed to be due to the presence of mysterious dark energy which, at present, contributes about 70% of the total universe energy–mass. What this dark energy is and where it came from is one of the fundamental problems of modern theoretical cosmology (for a recent review, see [1]). Assuming a constant equation of state (EOS), $p = w\rho$, the dark energy may be associated with a (so far unobserved) strange ideal fluid with negative w. Astrophysical data indicate that w lies in a very narrow strip close to $w = -1$. The case $w = -1$ corresponds to the cosmological constant. For w less than -1 the phantom dark energy is observed, and for w more than -1 (but less than $-1/3$) the dark energy is described by quintessence. It is interesting that the phantom phase is twice

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as probable as the quintessence phase. Moreover, there are indications that there occurred a recent transition over a cosmological constant barrier (over the phantom divide). In this case, which is not completely confirmed yet, the universe lives in its phantom phase which ends eventually at a future singularity (big rip) [2].

There are various approaches to describe the phantom dark energy. The simplest one is to work with negative kinetic energy scalars (phantoms). There may be several scalars in the theory, with at least one of them being phantom. The presence of a scalar potential or a non-minimal coupling with gravity is also necessary. The scalar phantom cosmology has been studied in a number of works (for recent discussion see [3–6]). The typical property of this phantom cosmology is a future singularity which occurs in a finite time. This is due to the growth of phantom energy which may lead to quite spectacular consequences. In particular, with the growth of phantom energy, typical energies (as well as curvature invariants) increase in the expanding universe. As a result, in some scenarios the quantum gravity era may come back at the end of the phantom universe evolution. In this case, it was checked that quantum effects [6–8] may act against the big rip (or even stop it in the case of quantum gravity [6]). In principle, there are four types of future singularities which are classified in [8].

The emerging phantom era may exist in string-inspired gravity where typically a scalar field as well as a Gauss–Bonnet term coupled to a scalar are included [9]. String considerations may also lead to effective (phantom-like) theories [10]. Finally, gravitational theory with a time-dependent cosmological constant [11] may also mimic the phantom regime.

An interesting approach to describe the dark energy universe is related to an ideal fluid with some (strange but explicit) EOS which may be sufficiently complicated [12]. Of course, this approach is phenomenological in some sense, because it does not describe the fundamental origin of dark energy. At the same time, it may lead to quite a successful description of not only the phantom phase but also of the transition from deceleration to acceleration or crossing of the phantom divide. Moreover, there is no need to introduce scalars in cosmology. One can generalize it assuming inhomogeneous EOS of the universe [13] which may be interpreted also as some modification of GR. In the present work, we discuss two dark energy models (one with implicit EOS of the universe and the other where gravity is modified by the function of the Gauss–Bonnet (GB) term). It is shown that such models lead to successful (quintessence, cosmological constant or phantom) late-time acceleration.

2. The acceleration due to dark energy with implicit EOS

We now consider the FRW cosmology with an ideal fluid. The starting FRW universe metric is

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^{3} (dx^i)^2. \tag{1}$$

In the FRW universe, the energy conservation law can be expressed as

$$0 = \dot{\rho} + 3H(p + \rho). \tag{2}$$

Here ρ is the energy density and p is the pressure. The Hubble rate H is defined by $H \equiv \dot{a}/a$ and the first FRW equation is

$$\frac{3}{\kappa^2} H^2 = \rho. \tag{3}$$

We often consider the case that ρ and p satisfy the simple EOS, $p = w\rho$. Then, if w is a constant, equation (2) can be easily integrated as $\rho = \rho_0 a^{-3(1+w)}$. Using the first FRW equation (3), the well-known solution follows:

$$a = a_0(t - t_1)^{\frac{2}{3(w+1)}} \qquad \text{or} \qquad a_0(t_2 - t)^{\frac{2}{3(w+1)}}, \tag{4}$$

when $w \neq -1$, and

$$a = a_0\, e^{\kappa t \sqrt{\frac{\rho_0}{3}}} \tag{5}$$

when $w = -1$.

The ideal fluid with a more general EOS may be considered [12]:

$$f(\rho, p) = 0. \tag{6}$$

An interesting example is given by

$$\rho\left(1 + \frac{A}{2}(\rho + p)\right) + \frac{3}{\rho}\left(2 + \frac{A}{2}(\rho + p)\right)^2 = 0. \tag{7}$$

Solving equations (2) and (3) one arrives at

$$H = \frac{1}{t}\left(1 - \frac{\kappa^2 t^2}{A}\right), \qquad \rho = \frac{3}{\kappa^2 t^2}\left(1 - \frac{\kappa^2 t^2}{A}\right)^2,$$
$$p = \frac{1}{\kappa^2}\left(\frac{1}{t^2} - \frac{8\kappa^2}{A} + \frac{3\kappa^4 t^2}{A^4}\right). \tag{8}$$

Since

$$\dot{H} = -\frac{1}{t^2} - \frac{\kappa^2}{A}, \tag{9}$$

it follows that

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = \frac{\kappa^2}{A}\left(-3 + \frac{\kappa^2 t^2}{A}\right). \tag{10}$$

Then if $A > 0$, the decelerating universe with $\ddot{a} < 0$ transits to an accelerating universe with $\ddot{a} > 0$ when $t = \sqrt{\frac{3A}{\kappa^2}}$. Equation (9) also shows that if $A < 0$, the non-phantom phase with $\dot{H} < 0$ changes to the phantom phase with $\dot{H} > 0$ at $t = \sqrt{-\frac{A}{\kappa^2}}$. This demonstrates that an ideal fluid with a complicated EOS may be the origin of dark energy and late-time acceleration.

3. Dark energy from modified Gauss–Bonnet gravity

In this section, we consider the following gravitational action [14]:

$$S = \int d^4x \left(\frac{1}{2\kappa^2}R + f(G) + \mathcal{L}_m\right). \tag{11}$$

Here $\mathcal{L}_m$ is the matter Lagrangian density and G is the GB invariant: $G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\xi\sigma}R^{\mu\nu\xi\sigma}$. Such a theory has a nice Newtonian limit and may be considered as an alternative for GR [14]. The variation over $g_{\mu\nu}$ gives

$$\begin{aligned}
0 = {} & \frac{1}{2\kappa^2}\left(-R^{\mu\nu} + \frac{1}{2}g^{\mu\nu}R\right) + T^{\mu\nu} + \frac{1}{2}g^{\mu\nu}f(G) - 2f'(G)RR^{\mu\nu} + 4f'(G)R^\mu_{\ \rho}R^{\nu\rho} \\
& - 2f'(G)R^{\mu\rho\sigma\tau}R^\nu_{\ \rho\sigma\tau} - 4f'(G)R^{\mu\rho\sigma\nu}R_{\rho\sigma} + 2(\nabla^\mu\nabla^\nu f'(G))R \\
& - 2g^{\mu\nu}(\nabla^2 f'(G))R - 4(\nabla_\rho\nabla^\mu f'(G))R^{\nu\rho} - 4(\nabla_\rho\nabla^\nu f'(G))R^{\mu\rho} \\
& + 4(\nabla^2 f'(G))R^{\mu\nu} + 4g^{\mu\nu}(\nabla_\rho\nabla_\sigma f'(G))R^{\rho\sigma} + 4(\nabla_\rho\nabla_\sigma f'(G))R^{\mu\rho\nu\sigma}. \tag{12}
\end{aligned}$$

The analogue of the first FRW equation has the following form:

$$0 = -\frac{3}{\kappa^2} H^2 + G f'(G) - f(G) - 24\dot{G} f''(G) H^3 + \rho_m. \tag{13}$$

Here ρ_m is the matter energy density. When $\rho_m = 0$, equation (13) has a de Sitter universe solution where H and therefore G are constants. If $H = H_0$ with constant H_0, equation (13) looks like [14]

$$0 = -\frac{3}{\kappa^2} H_0^2 + 24 H_0^4 f'\left(24 H_0^4\right) - f\left(24 H_0^4\right). \tag{14}$$

For a large number of choices of the function $f(G)$, equation (14) has a non-trivial ($H_0 \neq 0$) real solution for H_0 (de Sitter universe).

We now consider the case $\rho_m \neq 0$. Let the EOS parameter $w \equiv p_m/\rho_m$ be a constant. Using the conservation of energy, $\dot{\rho}_m + 3H(\rho_m + p_m) = 0$, one finds $\rho = \rho_0 a^{-3(1+w)}$. Assume $f(G)$ is given by

$$f(G) = f_0 |G|^\beta, \tag{15}$$

with constants f_0 and β. If $\beta < 1/2$, the $f(G)$ term becomes dominant compared with the Einstein term when the curvature is small. In this case, the solution is

$$a = \begin{cases} a_0 t^{h_0} & \text{when} \quad h_0 > 0 \\ a_0 (t_s - t)^{h_0} & \text{when} \quad h_0 < 0, \end{cases} \tag{16}$$

where

$$h_0 = \frac{4\beta}{3(1+w)},$$
$$a_0 = \left[-\frac{f_0(\beta - 1)}{(h_0 - 1)\rho_0} \left\{ 24 \left| h_0^3(-1 + h_0) \right| \right\}^\beta (h_0 - 1 + 4\beta) \right]^{-\frac{1}{3(1+w)}}. \tag{17}$$

One may define the EOS parameter w_{eff} by

$$w_{\text{eff}} = \frac{p}{\rho} = -1 - \frac{2\dot{H}}{3H^2}, \tag{18}$$

which is less than -1 if $\beta < 0$ and $w > -1$ as

$$w_{\text{eff}} = -1 + \frac{2}{3h_0} = -1 + \frac{1+w}{2\beta}. \tag{19}$$

If $\beta < 0$, we obtain the effective phantom phase with negative h_0 even if $w > -1$. In the phantom phase, the big rip type singularity at $t = t_s$ might occur. Near the big rip singularity, however, the curvature becomes dominant and $f(G)$-term may be neglected. Then the universe expands as $a = a_0 t^{2/3(w+1)}$. Hence, the big rip singularity eventually does not occur.

A similar model has been constructed in [15] within a consistent modified $f(R)$-gravity [16]. In the case of $f(R)$-gravity, instabilities appear in general [17]. Such instabilities are not common in $f(G)$-gravity.

We should note that under assumption (16), the GB invariant G and scalar curvature R behave as

$$G = \frac{24 h_0^3 (h_0 - 1)}{t^4} \qquad \text{or} \qquad \frac{24 h_0^3 (h_0 - 1)}{(t_s - t)^4},$$
$$R = \frac{6 h_0 (2h_0 - 1)}{t^2} \qquad \text{or} \qquad \frac{6 h_0 (2h_0 - 1)}{(t_s - t)^2}. \tag{20}$$

When the scalar curvature R is small, that is, t or $t_s - t$ is large, the GB invariant G becomes smaller more rapidly than R. When R is large, that is, t or $t_s - t$ is small, G becomes larger more

rapidly than R. Hence, if $f(G)$ is given by (15) with $\beta < 1/2$, the $f(G)$-term in the action (11) becomes dominant for small curvature but becomes less dominant for large curvature. Equation (17) follows when the curvature is small. There are, however, some exceptions. As clear from (20), when $h_0 = -1/2$, which corresponds to $w_{\text{eff}} = -7/3$, R vanishes and when $h_0 = -1$, corresponding to $w_{\text{eff}} = -5/3$, G vanishes. In these cases, only one of the Einstein and $f(G)$-terms survives.

One may also consider the case that $0 < \beta < 1/2$. As β is positive, the universe does not reach the phantom phase. When the curvature is strong, the $f(G)$-term could be neglected. If w is positive, the matter energy density ρ_m should behave as $\rho_m \sim t^{-2}$ but $f(G)$ behaves as $f(G) \sim t^{-4\beta}$. Thus, at a late time (large t), the $f(G)$-term dominates. Without the contribution from matter, equation (13) has a de Sitter universe solution where H and therefore G are constants. If $H = H_0$ with constant H_0, equation (13) looks like (14). Hence, even if one starts from the deceleration phase with $w > -1/3$, we may reach the asymptotically de Sitter universe, which is an accelerated universe. In other words, there occurs a transition from deceleration to acceleration in the universe evolution.

Now we consider the case where the contributions from the Einstein term and matter may be neglected. Equation (13) reduces to

$$0 = Gf'(G) - f(G) - 24\dot{G}f''(G)H^3. \tag{21}$$

If $f(G)$ behaves as (15), by assumption (16), it follows

$$0 = (\beta - 1)h_0^6(h_0 - 1)(h_0 - 1 + 4\beta). \tag{22}$$

As $h_0 = 1$ implies $G = 0$, we may choose $h_0 = 1 - 4\beta$. Equation (18) gives $w_{\text{eff}} = -1 + 2/3(1 - 4\beta)$. Therefore if $\beta > 0$, the universe accelerates ($w_{\text{eff}} < -1/3$) and if $\beta > 1/4$, the universe is in the phantom phase ($w_{\text{eff}} < -1$). The following model may also be constructed:

$$f(G) = f_i|G|^{\beta_i} + f_l|G|^{\beta_l}. \tag{23}$$

Here it is assumed

$$\beta_i > \tfrac{1}{2}, \qquad \tfrac{1}{2} > \beta_l > \tfrac{1}{4}. \tag{24}$$

When the curvature is large, as in the primordial universe, the first term dominates and

$$-1 > w_{\text{eff}} = -1 + \frac{2}{3(1 - 4\beta_i)} > -5/3. \tag{25}$$

On the other hand, when the curvature is small, as in the present universe, the second term in (23) dominates and gives

$$w_{\text{eff}} = -1 + \frac{2}{3(1 - 4\beta_l)} < -5/3. \tag{26}$$

Thus the modified GB gravity (23) may describe the unification of the inflation and the late-time acceleration of the universe (compare with proposal [18]).

Instead of (24), one may choose β_l as $\tfrac{1}{4} > \beta_l > 0$, which gives $-\tfrac{1}{3} > w_{\text{eff}} > -1$. Then the effective quintessence follows. By properly adjusting the couplings f_i and f_l in (23), one may obtain a period where the Einstein term dominates and the universe is in a deceleration phase. After that, a transition from deceleration to acceleration occurs when the GB term dominates.

Let us consider system (23) coupled with matter as in (13). We now choose $\beta_i > \tfrac{1}{2} > \beta_l$. When the curvature is large, as in the primordial universe, the first term dominates compared with the second and the Einstein terms. When the curvature is small, as in the present universe, the second term in (23) dominates compared with the second and the Einstein terms. The

effective w_{eff} is given by (19). In the primordal universe, the matter was radiation with $w = 1/3$, and then the effective w is given by $w_{i,\text{eff}} = -1 + 2/3\beta_i$, which could be less than $-1/3$. That is, the universe accelerates when $\beta_i > 1$. On the other hand, in the late-time universe matter could be dust with $w = 0$, and $w_{l,\text{eff}} = -1 + \frac{1}{2\beta_l}$, which is larger than 0 if $0 < \beta_l < 1/2$ or less than -1 if β_l is negative. Hence, in order that the acceleration of the universe could occur in both the primordial and late-time universe, the conditions $\beta_i > 1$ and $\beta_l < 0$ are required. In the same way, other specific models of a late-time accelerating universe may be constructed in frames of modified GB gravity without use of scalar fields.

The modified GB gravity cosmology remains to be studied in more detail, comparing it with observational data (in the same style as in [19]), investigating the perturbation structure in analogy with such study in other modified gravities [20]. One has to compare carefully the predictions of modified GB gravity with solar system tests in order to find the conditions on the form of function $f(G)$. Finally, for specific theories which admit the phantom phase where a final singularity may occur, a quantization programme should be developed (for recent related discussion, see [21]).

Acknowledgment

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